1. Motivation	Main definition	Finite index subgroups	A Gromov-like theorem	5. Generalizations
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The degree of commutativity of an infinite group

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GAGTA-9, Lumini.

September 17th, 2015.

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Outline				



- 2 Main definition
- Finite index subgroups
- A Gromov-like theorem



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- 2 Main definition
- Finite index subgroups
- 4 Gromov-like theorem
- 5 Generalizations

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Motivatio	on			

Theorem (Gustafson, 1973)

Let G be a finite group. If the probability that two elements from G commute is bigger than 5/8, then G is abelian.

$$dc(G) = rac{|\{(u,v) \in G^2 \mid uv = vu\}|}{|G|^2} = rac{1}{|G|^2} \sum_{u \in G} |C_G(u)| =$$

$$=rac{1}{|G|^2}\Big(|Z(G)||G|+\sum_{u\in G\setminus Z(G)}|C_G(u)|\Big)\leqslant$$

$$\leqslant rac{1}{|G|^2}\left(|Z(G)||G|+(|G|-|Z(G)|)rac{|G|}{2}
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Motivatio	าท			

Theorem (Gustafson, 1973)

Let G be a finite group. If the probability that two elements from G commute is bigger than 5/8, then G is abelian.

Proof. Suppose G is not abelian. Then,

$$dc(G) = \frac{|\{(u,v) \in G^2 \mid uv = vu\}|}{|G|^2} = \frac{1}{|G|^2} \sum_{u \in G} |C_G(u)| = \frac{1}{|G|^2} \left(|Z(G)||G| + \sum_{u \in G \setminus Z(G)} |C_G(u)| \right) \leq \frac{1}{|G|^2} \left(|Z(G)||G| + (|G| - |Z(G)|) \frac{|G|}{2} \right) =$$

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= $\frac{|G| + |Z(G)|}{2|G|} \le \frac{1}{2} + \frac{|G|}{4 \cdot 2|G|} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8},$
cause $G/Z(G)$ cannot be cyclic and so, $|Z(G)| \le |G|/4.$

The quaternion group has dc(Q) = 5/8.

"There is no live between 5/8 and 1"

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Degree of commutativity							

Let $G = \langle X \rangle$ be a f.g. group. The degree of commutativity of G w.r.t. X is

$$dc_X(G) = \limsup_{n \to \infty} \frac{|\{(u, v) \in \mathbb{B}_X(n) \times \mathbb{B}_X(n) \mid uv = vu\}|}{|\mathbb{B}_X(n)|^2} \in [0, 1],$$

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where
$$\mathbb{B}_X(n) = \{g \in G \mid |g|_X \leqslant n\}.$$

Question

Is this a real lim ? Does it depend on X ?

- No example where lim doesn't exist;
- No proof it is always a real limit.

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- subexponential growth if $\lim_{n\to\infty} \frac{|\mathbb{B}_X(n+1)|}{|\mathbb{B}_X(n)|} = 1$;
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Independence on X

Definition

A f.g. group $G = \langle X \rangle$ is of

- subexponential growth if $\lim_{n\to\infty} \frac{|\mathbb{B}_X(n+1)|}{|\mathbb{B}_X(n)|} = 1$;
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Definition

Let $G = \langle X \rangle$. A map $f : G \to \mathbb{N}$ is an estimation of the X-metric if $\exists K > 0$ such that $\forall w \in G$

$$\frac{1}{K}f(w)\leqslant |w|_X\leqslant Kf(w).$$

Example

It is well known that, for $G = \langle X \rangle = \langle Y \rangle$, $| \cdot |_X$ is an estimation of the *Y*-metric, and $| \cdot |_Y$ is an estimation of the *X*-metric.

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Define the *f*-ball and the *f*-dc:

 $\mathbb{B}_f(n) = \{ w \in G \mid f(w) \leqslant n \},$

 $dc_f(G) = \limsup_{n \to \infty} \frac{|\{(u, v) \in \mathbb{B}_f(n) \times \mathbb{B}_f(n) \mid uv = vu\}|}{|\mathbb{B}_f(n)|^2}$

Proposition

Let $G = \langle X \rangle$ be of polynomial growth, and $f : G \to \mathbb{N}$ be an estimation of the X-metric. Then,

 $dc_X(G) > 0 \iff dc_f(G) > 0.$

Proof. Clearly, $\mathbb{B}_f(n) \subseteq \mathbb{B}_X(Kn) \subseteq \mathbb{B}_f(K^2n)$ so, $|\{(u, v) \in (\mathbb{B}_f(n))^2 \mid uv = vu\}| \leq |\{(u, v) \in (\mathbb{B}_X(Kn))^2 \mid uv = vu\}|.$

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Corollary

If $G = \langle X \rangle = \langle Y \rangle$ is of polynomial growth, then

 $dc_X(G) = 0 \iff dc_Y(G) = 0.$

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Let $\langle Y \rangle = H \leq G = \langle X \rangle$. The subgroup H is undistorted if $\exists K > 0$ s.t. $\forall h \in H$, $|h|_Y/K \leq |h|_X \leq K|h|_Y$. In this case, $|\cdot|_X$ restricted to H is an estimation of the Y-metric for H.

Corollary

Let $G = \langle X \rangle$ be of polynomial growth, and $\langle Y \rangle = H \leqslant G$ be a non-distorted subgroup. Then,

 $dc_X(H) = 0 \iff dc_Y(H) = 0.$

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- Finite index subgroups
- A Gromov-like theorem





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Finite index subgroups

Lemma (Burillo-Ventura, 2002)

If $H \leq_{f.i.} G = \langle X \rangle$ and G has subexponential growth then, for every $g \in G$, there exists $\lim_{n \to \infty} \frac{|\mathbb{B}_X(n) \cap gH|}{|\mathbb{B}_X(n)|} = \lim_{n \to \infty} \frac{|\mathbb{B}_X(n) \cap Hg|}{|\mathbb{B}_X(n)|} = \frac{1}{[G:H]}$.

Remark

This is false in the free group: $H = \{even words\} \leq_2 F_r$.

Proposition

Let $\langle Y \rangle = H \leq_{f.i.} G = \langle X \rangle$ be of polynomial growth. Then,

$$dc_X(G) \ge \frac{1}{[G:H]^2} dc_X(H).$$

In particular, $dc_Y(H) > 0 \Rightarrow dc_X(H) > 0 \Rightarrow dc_X(G) > 0$.

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Finite in	dex subgr			

Proof. Clearly, $|\{(u, v) \in (\mathbb{B}_{X}(n))^{2} \mid uv = vu\}| \ge |\{(u, v) \in (H \cap \mathbb{B}_{X}(n))^{2} \mid uv = vu\}|.$

 Motivation 	Main definition	Finite index subgroups	A Gromov-like theorem	Generalizations			
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Finite index subgroups

Proof. Clearly, $|\{(u, v) \in (\mathbb{B}_{X}(n))^{2} \mid uv = vu\}| \ge |\{(u, v) \in (H \cap \mathbb{B}_{X}(n))^{2} \mid uv = vu\}|.$ Therefore, given $\varepsilon > 0$, we have for $n \gg 0$ $\frac{|\{(u,v)\in (\mathbb{B}_X(n))^2 \mid uv = vu\}|}{|\mathbb{B}_X(n)|^2} \ge$ $\frac{|\{(u,v)\in (H\cap\mathbb{B}_X(n))^2\mid uv=vu\}|}{|H\cap\mathbb{B}_X(n)|^2}\cdot\frac{|H\cap\mathbb{B}_X(n)|^2}{|\mathbb{B}_X(n)|^2}\geq$ $\frac{|\{(u,v)\in (H\cap\mathbb{B}_X(n))^2\mid uv=vu\}|}{|H\cap\mathbb{B}_X(n)|^2}\left(\frac{1}{[G:H]}-\varepsilon\right)^2,$

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Finite index subgroups

Proof. Clearly, $|\{(u, v) \in (\mathbb{B}_{X}(n))^{2} \mid uv = vu\}| \ge |\{(u, v) \in (H \cap \mathbb{B}_{X}(n))^{2} \mid uv = vu\}|.$ Therefore, given $\varepsilon > 0$, we have for $n \gg 0$ $\frac{|\{(u,v)\in (\mathbb{B}_X(n))^2 \mid uv = vu\}|}{|\mathbb{B}_X(n)|^2} \ge$ $\frac{|\{(u,v)\in (H\cap\mathbb{B}_X(n))^2\mid uv=vu\}|}{|H\cap\mathbb{B}_X(n)|^2}\cdot\frac{|H\cap\mathbb{B}_X(n)|^2}{|\mathbb{B}_X(n)|^2}\geq$ $\frac{|\{(u,v)\in (H\cap\mathbb{B}_X(n))^2\mid uv=vu\}|}{|H\cap\mathbb{B}_X(n)|^2}\left(\frac{1}{[G:H]}-\varepsilon\right)^2,$ Taking limsups, $dc_X(G) \ge dc_X(H) \left(\frac{1}{[G:H]} - \varepsilon\right)^2$. And this is true

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Finite index subgroups

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Finite index subgroups						
1. Motivation	2. Main definition	 Finite index subgroups OO●O 	 A Gromov-like theorem 0000 	5. Generalizations		

Let G be a finite group and $H \leq G$. Then, $dc(G) \leq dc(H) \cdot dc(G/H)$.

Proposition

Let $G = \langle X \rangle$ be subexponentially growing. Then, for any finite quotient G/N, we have $dc_X(G) \leq dc(G/N)$.

Proof. Let $N \trianglelefteq G$ with [G : N] = d. By B-V, $\forall g \in G \lim_{n \to \infty} |gN \cap \mathbb{B}_X(n)| / |\mathbb{B}_X(n)| = 1/d$, indep. X and g. But $|G/N| < \infty$, so this lim is uniform on g, i.e., $\forall \varepsilon > 0 \exists n_0 \forall n \ge n_0$ and $\forall g \in G$,

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Finite index subgroups						
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 $\exists \delta > 0 \text{ s.t. } |\{(u, v) \in (\mathbb{B}_X(n))^2 \mid uv = vu\}|/|\mathbb{B}_X(n)|^2 > dc(G/N) + \delta$ for infinitely many n's.

 $2\varepsilon d + \varepsilon^2 d^2 \le \delta$, and $\exists n \gg 0$ such that

$$dc(G/N) + \delta < \frac{|\{(u,v) \in (\mathbb{B}_X(n))^2 \mid uv = vu\}|}{|\mathbb{B}_X(n)|^2}$$

$$\leq \frac{1}{|\mathbb{B}_X(n)|^2} \left| \left\{ (\overline{u}, \overline{v}) \in (G/N)^2 \mid \overline{u} \, \overline{v} = \overline{v} \, \overline{u} \right\} \right| \, \left(\frac{1}{d} + \varepsilon \right)^2 |\mathbb{B}_X(n)|^2$$

$$=\frac{|\{(\overline{u},\overline{v})\in (G/N)^2\mid \overline{u}\,\overline{v}=\overline{v}\,\overline{u}\}|}{d^2}(1+\varepsilon d)^2$$

$$\leqslant \frac{|\{(\overline{u},\overline{v})\in (G/N)^2\mid \overline{u}\,\overline{v}=\overline{v}\,\overline{u}\}|}{d^2}+2\varepsilon d+\varepsilon^2 d^2$$

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Fir	nite index subgrou	ups		
	$\exists \delta > 0 \text{ s.t. } \{ (u, v) \in (\mathbb{B} \ for infinitely many n's.$ In the above inequality, $2\varepsilon d + \varepsilon^2 d^2 \leqslant \delta$, and $\exists n$	$_X(n))^2 \mid uv = vu\}$ take $arepsilon > 0$ small ϵ $\gg 0$ such that	$ / \mathbb{B}_X(n) ^2 > dc(G/N)$ enough so that	$l) + \delta$
	dc(G/N) + c	$\delta < \frac{ \{(u,v) \in (\mathbb{B}_X \ \ \ \ \ \ \ \ }{\ \ \ \ \ \ \ \ \ \ \ $	$(n))^2 uv = vu \} $ $(n) ^2$	
	$\leqslant \frac{1}{ \mathbb{B}_X(n) ^2} \{(\overline{u},\overline{v})$	$\in (G/N)^2 \mid \overline{u}\overline{v} = 1$	$\overline{v}\overline{u}\} \left(\frac{1}{d}+\varepsilon\right)^2 \mathbb{B}_X($	[<i>n</i>) ²
	$= \frac{ \{(\overline{u},\overline{v})\} }{ \overline{u} }$	$\frac{1}{e} (G/N)^2 \mid \overline{u} \overline{v} = \frac{1}{2}$	$\overline{\overline{v}\overline{u}\}} (1+\varepsilon d)^2$	
		$(G/N)^2 \overline{u}\overline{v} - \overline{v}$		

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Finite index subgroups					

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Finite index subaroups						
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 $\exists \delta > 0 \text{ s.t. } |\{(u, v) \in (\mathbb{B}_X(n))^2 \mid uv = vu\}|/|\mathbb{B}_X(n)|^2 > dc(G/N) + \delta$ for infinitely many n's. In the above inequality, take $\varepsilon > 0$ small enough so that $2\varepsilon d + \varepsilon^2 d^2 \leq \delta$, and $\exists n \gg 0$ such that

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Finite index subaroups						
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The mai	n result			

Let $G = \langle X \rangle$ be of subexponential growth and residually finite. Then,

(i) $dc_X(G) > 5/8 \Rightarrow G$ is abelian;

(ii) $dc_X(G) > 0 \Leftrightarrow G$ is virtually abelian.

In particular, (i) and (ii) is true for polynomially growing groups.

Corollary

Let $G = \langle X \rangle = \langle Y \rangle$ be of subexponential growth and residually finite. Then,

 $dc_X(G)=0 \quad \Longleftrightarrow \quad dc_Y(G)=0.$

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 $dc_X(G)=0 \quad \Longleftrightarrow \quad dc_Y(G)=0.$

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The mai	n result			

Let $G = \langle X \rangle$ be of subexponential growth and residually finite. Then,

(i) $dc_X(G) > 5/8 \Rightarrow G$ is abelian;

(ii) $dc_X(G) > 0 \Leftrightarrow G$ is virtually abelian.

In particular, (i) and (ii) is true for polynomially growing groups.

Corollary

Let $G = \langle X \rangle = \langle Y \rangle$ be of subexponential growth and residually finite. Then,

 $dc_X(G) = 0 \iff dc_Y(G) = 0.$

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Coniect	ure			

Conjecture

For any finitely generated group $G = \langle X \rangle$,

 $dc_X(G) > 0 \iff G$ is virtually abelian.

Conjecture

Every finitely generated group G with super-polynomial growth has $dc_X(G) = 0$ for every X.

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Conjecti	ure			

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 Motivation 	Main definition	Finite index subgroups	A Gromov-like theorem	Generalizations
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Proof c	of the main	rocult		

Let $G = \langle X \rangle$ be of subexponential growth and residually finite. Then, (i) $dc_X(G) > 5/8 \Rightarrow G$ is abelian; (ii) $dc_X(G) > 0 \Leftrightarrow G$ is virtually abelian.

Proof. (i). Suppose $dc_X(G) > 5/8$. Then, dc(G/N) > 5/8 for every $N \leq_{f.i.} G$. Hence, by Gustafson's thm, every finite quotient of G is abelian. Residual finiteness implies G abelian.

(ii, \Leftarrow). Suppose $G = \langle X \rangle$ is virtually abelian, $\langle Y \rangle = H \leq_{f.i.} G$ with H abelian. Then G is polynomially growing and $dc_Y(H) = 1 > 0$ so, $dc_X(G) > 0$.

 Motivation 	Main definition	Finite index subgroups	A Gromov-like theorem	Generalizations	
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Proof of	the main	rocult		

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Proof of	the main	result		
1. Motivation	2. Main definition	 Finite index subgroups OOOO 	 A Gromov-like theorem OOO● 	5. Generalizations

Claim. If H is f.g., r.f., not virtually abelian then $\exists K \leq_{ch.} H$ such that H/K is (finite) not abelian.

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 1. Motivation
 2. Main definition
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$$K_2 \leq_{ch., f.i.} K_1 \leq_{ch., f.i.} K_0 = G,$$

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$$\cdots \trianglelefteq_{\stackrel{f_i}{f_i}} K_i \trianglelefteq_{\stackrel{f_i}{f_i}} K_{i-1} \trianglelefteq_{\stackrel{f_i}{f_i}} \cdots \oiint_{\stackrel{f_i}{f_i}} K_2 \trianglelefteq_{\stackrel{ch_i}{f_i}} K_1 \trianglelefteq_{\stackrel{ch_i}{f_i}} K_0 = G,$$

1. Motivation 2. Main definition 3. Finite index subgroups 4. A Gromov-like theorem 5. Generalizations 00 0000 0000 0000 0000 0000

Claim. If H is f.g., r.f., not virtually abelian then $\exists K \leq_{ch.} H$ such that H/K is (finite) not abelian.

$$\cdots \trianglelefteq_{ch, K_i} K_i \trianglelefteq_{ch, K_{i-1}} \bowtie_{f, i} \cdots \bowtie_{ch, K_2} \bowtie_{f, i} K_1 \bowtie_{ch, K_0} = G,$$

such that K_{i-1}/K_i is not abelian so, $dc(K_{i-1}/K_i) \leq 5/8 \quad \forall i$.

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Claim. If H is f.g., r.f., not virtually abelian then $\exists K \leq_{ch.} H$ such that H/K is (finite) not abelian.

$$\begin{split} & \cdots \trianglelefteq_{ch., K_i} \leq_{ch., K_{i-1}} \leq_{ch., \cdots} \leq_{ch., K_2} \leq_{ch., K_1} \leq_{ch., K_1} K_0 = G, \\ & \text{such that } K_{i-1}/K_i \text{ is not abelian so, } dc(K_{i-1}/K_i) \leqslant 5/8 \quad \forall i. \\ & \text{Then } \forall i, \quad K_i \trianglelefteq G, \quad (G/K_i)/(K_{i-1}/K_i) = G/K_{i-1} \text{ and, by Gallagher,} \end{split}$$

 $dc(G/K_i) \leqslant dc(K_{i-1}/K_i) \cdot dc(G/K_{i-1}) \leqslant 5/8 \cdot dc(G/K_{i-1}).$

1. Motivation 2. Main definition 3. Finite index subgroups 4. A Gromov-like theorem 5. Generalizations 00 0000 000 000 000 000 Proof of the main result 1. 1. 1. 1.

Claim. If H is f.g., r.f., not virtually abelian then $\exists K \leq_{ch.} H$ such that H/K is (finite) not abelian.

$$\begin{split} & \cdots \trianglelefteq_{ch_{i}} K_{i} \trianglelefteq_{ch_{i}} K_{i-1} \trianglelefteq_{ch_{i}} \cdots \trianglelefteq_{ch_{i}} K_{2} \trianglelefteq_{ch_{i}} K_{1} \trianglelefteq_{ch_{i}} K_{0} = G, \\ & \text{such that } K_{i-1}/K_{i} \text{ is not abelian so, } dc(K_{i-1}/K_{i}) \leqslant 5/8 \quad \forall i. \\ & \text{Then } \forall i, \quad K_{i} \trianglelefteq G, \quad (G/K_{i})/(K_{i-1}/K_{i}) = G/K_{i-1} \text{ and, by Gallagher,} \\ & dc(G/K_{i}) \leqslant dc(K_{i-1}/K_{i}) \cdot dc(G/K_{i-1}) \leqslant 5/8 \cdot dc(G/K_{i-1}). \\ & \text{By induction, } dc(G/K_{i}) \leqslant (5/8)^{i} \text{ and so,} \\ & \quad dc_{X}(G) \leqslant dc(G/K_{i}) \leqslant (5/8)^{i}, \end{split}$$

for every *i*. Therefore, $dc_X(G) = 0$. \Box

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Outline				

Motivation

- 2 Main definition
- 3 Finite index subgroups
- A Gromov-like theorem





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Genera	lizations			

• We can replace the uniform measures on balls to any sequence of measures (random walks, etc).

Definition

Let $\{X_1, \ldots, X_k\}$ be a set of abstract variables and \mathcal{F} the free group on it. Think elements $w \in \mathcal{F}$ as equations, w = 1, and subsets $\mathcal{E} \subseteq \mathcal{F}$ as systems of equations. Define solutions on a group G in the obvious way.

Definition

Given $G = \langle X \rangle$ and a system of equations $\mathcal{E} \subseteq \mathcal{F}$, we define the degree of satisfiability of \mathcal{E} in G as

 $ds_X(G,\mathcal{E}) = \limsup_{n \to \infty} \frac{|\{(g_1,\ldots,g_k) \in (\mathbb{B}_X(n))^k \mid (g_1,\ldots,g_k) \text{ sol. } \mathcal{E}\}|}{|\mathbb{B}_X(n)|^k} \in [0,1].$

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Genera	lizations			

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1. Motivation	Main definition	Finite index subgroups	4. A Gromov-like theorem	Generalizations

Definition

Let G and \mathcal{E} be as before. Fix a collection of measures μ_n in G with finite support, $|\text{Supp }\mu_n| < \infty$, and such that

 $\operatorname{Supp} \mu_1 \subseteq \operatorname{Supp} \mu_2 \subseteq \cdots$

and $\cup_{n \in \mathbb{N}}$ Supp $\mu_n = G$. We define the degree of satisfiability of \mathcal{E} in G w.r.t. μ_n as

 $ds_X(G, \mathcal{E}, \{\mu_n\}_n) =$

 $\limsup_{n\to\infty}\mu_n^{\times k}\big(\{(g_1,\ldots,g_k)\in G^k\mid (g_1,\ldots,g_k) \text{ sol. } \mathcal{E}\}\big)\in[0,1].$

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Conjecture

Let G, \mathcal{E} , and $\{\mu_n\}_n$ be as above, with μ_n "reasonable". Then,

 $ds(G, \mathcal{E}, {\mu_n}_n) > 0 \iff \mathcal{E}$ is a virtual law in G.

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 ${\mathcal E}$ is a law in G if every $(g_1,\ldots,g_k)\in G^k$ is a solution of ${\mathcal E}$ in G.

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THANKS